

## Natural fueling of a tokamak fusion reactor

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A natural fueling mechanism that helps to maintain the main core deuterium and tritium (DT) density profiles in a tokamak fusion reactor is presented. In  $H$ -mode plasmas dominated by ion-temperature gradient (ITG) driven turbulence, cold DT ions near the edge will naturally pinch radially inward toward the core. This mechanism is due to the quasineutral heat flux dominated nature of ITG turbulence and still applies when trapped and passing kinetic electron effects are included. Fueling using shallow pellet injection is augmented by an inward pinch of cold DT fuel. The natural fueling mechanism is investigated using the gyrokinetic turbulence code GEM [Y. Chen and S. E. Parker, *J. Comput. Phys.* **220**, 839 (2007)] and is analyzed using quasilinear theory. Profiles similar to those used for conservative International Thermonuclear Experimental Reactor [R. Aymar *et al.*, *Nucl. Fusion* **41**, 1301 (2001)] transport modeling that have a completely flat density profile are examined and it is found that natural fueling actually reduces the linear growth rate and energy transport. Natural fueling requires a two-component plasma and ion-ion and charge-exchange collisions set limits on this favorable effect. © 2010 American Institute of Physics. [doi:10.1063/1.3389225]

The fueling of a tokamak fusion reactor is an important scientific and technological problem.<sup>1–3</sup> In International Thermonuclear Experimental Reactor (ITER), deep central fueling is somewhat uncertain using present-day high-field side pellet injection schemes.<sup>4</sup> In a tokamak with deuterium, tritium, and helium ash ion species, above certain level of density concentration the helium ash naturally diffuses outward,<sup>5</sup> helping in ash removal. Additionally, due to quasineutrality, the main deuterium and tritium (DT) species go radially inward (or pinch) to balance the helium ash outward flux. In this letter, we show how adding cold fuel toward the edge can fuel the core and maintain the DT density profiles. Global multispecies gyrokinetic simulations using the GEM code,<sup>6</sup> show the effect that cold DT fuel added near the edge will naturally pinch toward the core. The main DT inward fluxes can be significantly reduced by fueling, while the outward flux of helium ash is maintained. The cold DT fuel will heat up due to ion-ion collisions as it moves toward the core, assuming the equilibration time is not too fast, the core density profiles may be sustainable with shallow pellet injection. We will show later that the transport time scale of the cold fuel is faster than the equilibration time in which it heats.

In earlier studies of the “ion-mixing mode,”<sup>7</sup> an inflow of both electrons and ions is generated when the ion-temperature gradient is steeper than the density gradient. Here, the results rely on an inward pinch of cold DT fuel and not a net inward particle flux. The results presented here may also help provide further insight on the nonadiabatic particle pinch<sup>8,9</sup> and recent theories of density peaking.<sup>10,11</sup>

We first present simulation results for the Cyclone base case,<sup>12,13</sup> which are parameters from a DIII-D (Ref. 14)  $H$ -mode assuming concentric circle flux-surface toroidal geometry. Here, we examine a *global* version of the case with kinetic electrons and in the electrostatic limit. There are five ion species: the main DT, the helium ash, and the DT fuel.

The major radius is  $R=445\rho_s$  and minor radius is  $a=160\rho_s$ , where  $\rho_s \equiv \sqrt{T_e/T_i}\rho_i$  and  $\rho_i$  is the thermal proton gyroradius at a reference temperature  $T_0$ . The  $q$  profile is  $q(r)=0.85+2.2 \times (r/a)^2$ . The density  $n(r)$  and temperature  $T(r)$  profiles, normalized to the reference density  $n_0$  and temperature  $T_0$ , are shown in Fig. 1. The main DT profiles are selected so that  $L_{n,T}^{-1}(r)=B_{n,T} \operatorname{sech}^2[(r-r_0)/w]$ , where  $L_n \equiv -n/(\partial n/\partial r)$ ,  $L_T \equiv -T/(\partial T/\partial r)$ ,  $r_0=a/2$ , and  $w=40\rho_s$ . The constants  $B_{n,T}$  are determined by requiring  $R/|L_n(r_0)|=2.2$  and  $R/|L_T(r_0)|=6.9$ . We assume the helium ash and electrons have the same temperature as the main DT. A linear density profile is assumed for the helium ash, as shown in the figure. The density profile of the DT fuel has the form  $n(r)=C\{1+\tanh[5(r/a-0.65)]\}/4$ , in which  $C$  denotes the density concentration, e.g.,  $C=0.05$  means the peak density of the cold DT fuel is 5% of peak density of main core DT. This cold fuel is chosen so that the peak of the fuel density is toward the edge of the simulation domain, at  $r/a=0.8$ . This is a multicomponent plasma with a *total* DT profile monotonically decreasing with minor radius throughout the simulation domain.

The temperature of the DT fuel is set to be constant and is equal to the temperature of the main species at the simulation boundary near to the edge. Hence, the DT fuel is cold compared to the main DT species. The three-dimensional simulation domain of  $0.15 \leq r/a \leq 0.85$  is discretized using a  $128 \times 64 \times 32$  grid. The time step is  $\Delta t=2\Omega_i^{-1}$ , where  $\Omega_i$  is the proton gyrofrequency at  $T_0$ . We use 4 194 304 particles per species with realistic mass ratios, e.g., the helium mass is  $4 \times 1837$  times of the electron mass. The simulation uses GEM, which is a global gyrokinetic  $\delta f$  particle-in-cell code.<sup>6,15</sup>

We study the effects of natural fueling by running simulations with different concentrations of the DT fuel density. Figure 2 shows the particle fluxes of the deuterium fuel, main deuterium, and the helium ash for concentrations from  $C=10^{-6}$  (no fueling) to  $C=0.1$ . The particle fluxes are calcu-

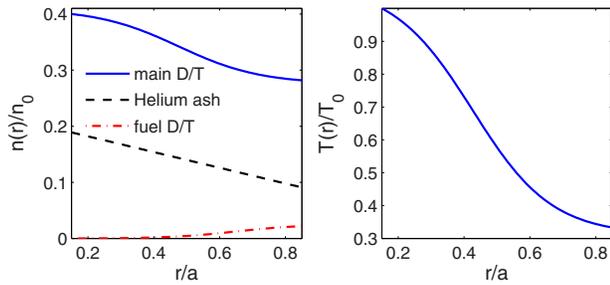


FIG. 1. (Color online) Density and temperature profiles of the Cyclone base case. Left: densities of main DT, helium ash, and the DT fuel; right: temperature of the main species.

lated as  $\Gamma = n_0/N \sum_j w_j v_{Ex}$ , where  $w_j$  is the weight of particle  $j$ ,  $v_{Ex}$  is the particle's  $\mathbf{E} \times \mathbf{B}$  drift velocity in the  $r$  direction, and  $N$  is total number of particles. The fluxes are saved at eight points along the minor radius  $r$ , and the results shown in Fig. 2 are volume averaged over all the points. The main and fueling tritium fluxes are similar to those of the deuterium for this case and not shown. From Fig. 2(a), the cold fuel flow always goes inward, and the level increases for bigger  $C$ . Hence, “natural fueling,” that is, the inward pinch of the cold fuel, is demonstrated for this simplified case.

Estrada-Mila *et al.*<sup>5</sup> predicts using both theory and gyrokinetic simulation that for  $L_{nHe}/L_{ne} < 0.84$ , the helium ash flow should go outward, and this condition is well satisfied for the density profiles presented here. For the results in this letter, the helium ash always goes out. Assuming nearly adiabatic electrons, consistent with the properties of ion-temperature gradient (ITG) turbulence, quasineutrality requires that the outgoing helium ash flow must generate some inward going DT flow. Without the DT fuel, i.e., for the

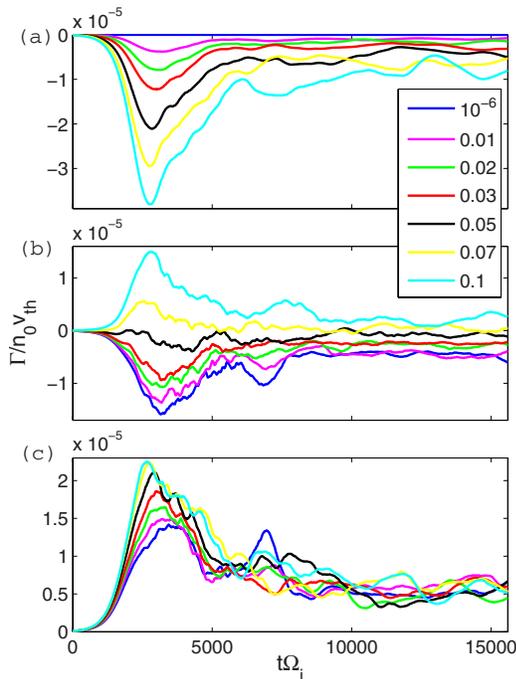


FIG. 2. (Color online) The Cyclone base case: the volume averaged particle fluxes of (a) the deuterium fuel, (b) the main deuterium, and (c) the helium ash for  $C=10^{-6}$ , 0.01, 0.02, 0.03, 0.05, 0.07, and 0.1. The flux is normalized to  $n_0 v_{th}$ , where  $v_{th}$  is the proton thermal velocity at  $T_0$ .

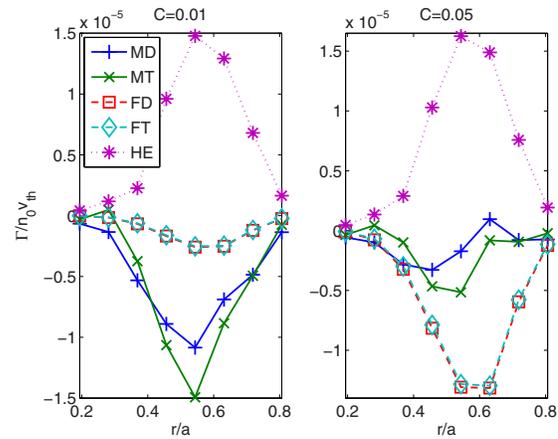


FIG. 3. (Color online) The time-averaged particle fluxes of the main deuterium (MD), main tritium (MT), fuel deuterium (FD), fuel tritium (FT), and helium ash (HE) along  $r$  for  $C=0.01$  and  $C=0.05$ .

$C=10^{-6}$  case, the particle fluxes of main DT go inward as the helium ash goes out. This phenomenon is clearly shown in Figs. 2(b) and 2(c). Therefore, edge fueling is needed to maintain the main DT density profile.

By adding the cold fuel from the edge, the inward particle flux of the main DT is reduced. As shown in Fig. 2(b), for  $C=0.05$ , the particle flux of the main deuterium is nearly zero. Now, the helium ash goes out at the expense of the cold DT fuel instead of the hot main DT. Natural fueling is demonstrated and the main DT profile is maintained. The cold DT fuel, of course, must be heated as it migrates toward the core.

This process is better demonstrated in Fig. 3, where the time-averaged particle fluxes of all five species are shown across the minor radius. Comparing the results of  $C=0.05$  to  $C=0.01$ , the increased DT fuel fluxes apparently cancel that of the main DT. We note that although this cancellation is significant, it is not complete. This is because the positions of the maximum flux are different for the main and fuel DT due to their individual density profiles. The cold fuel concentration density should be kept below a threshold value above which the main DT flux becomes radially outward. As in Fig. 2(b), for the case  $C=0.07$  and  $C=0.1$ , the main DT fluxes go outward in this case.

Figure 4 shows the effects of fueling on all species. After normalizing by their individual densities, it is clear to see that fueling is most effective at pinching DT fuel inward. The

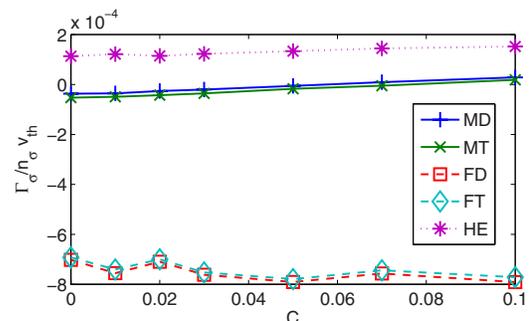


FIG. 4. (Color online) The density-normalized particle fluxes  $\Gamma_\sigma/n_\sigma v_{th}$  of the five species for different  $C$ 's.

particle flow of the main DT is altered from inward to near zero. The outgoing helium ash flow appears insensitive to fueling. This is important, in that the natural fueling does not increase helium ash build up.

The simulation results presented here show similar behavior as seen in pellet injection experiments where cold fuel moves radially inward on a millisecond time scale.<sup>16</sup> It is found in pellet injection experiments that as the pellet ablates, a cold plasma forms around the pellet aligned with the magnetic field. As the pellet moves inward, the cloud stays on the flux surface and expands mainly along the field line.<sup>16</sup> The equilibration time between the cold cloud ions and the bulk plasma ions was found to be five times that of the time for the fuel to move radially inward for Tore Supra and the tokamak fusion test reactor experimental parameters.<sup>17</sup> In this study, we simply assume the cold fuel has the same temperature as the background plasma at the edge. The cold plasma source from shallow pellet injection would likely be even colder.

For this natural fueling mechanism to work, the fuel should remain cold as it migrates toward the core. The fuel temperature will come in equilibrium with the core temperature on the ion-ion equilibration time. For a tokamak fusion reactor at 15 keV in the core such as ITER, assuming the cold fuel temperature is 2 keV, the ion-ion thermal equilibration time is about 0.03 s. From Fig. 4, the averaged flow velocity of the fuel is about  $7 \times 10^{-4} v_{th}$ , where  $v_{th}$  is the proton thermal velocity calculated at  $T_0$ . At fusion temperature,  $v_{th}$  is more than  $10^6$  m/s, therefore during the equilibration time, the fuel can go as far as 20 m, which is much larger than the minor radius. The strong inward flux velocity is not simply caused by the negative density gradient of the fuel but rather due to the cold temperature. In fact, a pinch exists for cold fuel even in simulations with no density gradient, indicating a convective pinch rather than diffusive density transport.

Next, a multispecies quasilinear theory is used to explain the natural inward pinch of the DT fuel. For simplicity, assume there are only two ion species: the hot main deuterium labeled as “i” and the cold fuel deuterium, as “I.” We extend the toroidal gyro-Landau fluid model of Ref. 18 to include multiple ion species. The linear dispersion relation is

$$R_i + R_I = 1 - i\delta \quad (1)$$

in which the  $i\delta$  denotes the nonadiabatic electron response

$$R_\sigma = f_\sigma \tau_\sigma \frac{(\omega_{*\sigma} - \omega_{D\sigma})(\omega + \omega_{D\sigma} y_i) - \omega_{D\sigma} \omega_{*p\sigma} + X^* \omega_{D\sigma}^2}{\omega(\omega + \omega_{D\sigma} y_i) - \omega_{D\sigma}^2 z_i}, \quad (2)$$

$\tau_\sigma = T_e / T_\sigma$ ,  $\sigma = i, I$ ,  $f_i = \epsilon$ ,  $f_I = 1 - \epsilon$ , and  $\epsilon \equiv n_I / n_e$ .  $X^* = 7/4$ ,  $z_i = -1$ , and  $y_i = X^* - z_i$  are parameters used in the gyro-Landau theory without dissipation. The magnetic drift frequency is taken in its  $\theta = 0$  value,  $\omega_{D\sigma} = 2(L_n / R) \omega_{*\sigma}$ , where  $\omega_{*\sigma} = k_\theta T_\sigma / (eBL_{n\sigma})$ ,  $\omega_{*T\sigma} = k_\theta T_\sigma / (eBL_{T\sigma})$ , and  $\omega_{*p\sigma} = \omega_{*\sigma} + \omega_{*T\sigma}$ . The quasilinear particle flux is then given by

$$\Gamma_\sigma = \text{Re}[ik_\theta \rho_s |\phi|^2 R_\sigma(\omega) n_e], \quad (3)$$

hence the direction of the fuel flux is determined by  $-\text{Im}(R_I)$ , with the inflow of the fuel corresponding to a negative  $\Gamma_I$ .

Equations (1) and (2) are solved using the Cyclone base case parameters and typical ITG value of  $k_\theta \rho_s = 0.4$ . For  $\epsilon = 0.05$  and  $\delta = 0.1$ , the resulting real frequency  $\omega_R = -0.0014 \Omega_i$ , which is negative and consistent with ITG instability, linear growth rate  $\gamma = 0.0011 \Omega_i$ , which is comparable to the simulation result of 0.00068;  $-\text{Im}(R_I) = -0.25$ , indicating a pinch for the cold fuel ion. For a wide range of  $\epsilon$ ,  $\delta$ , and  $\omega_{*I}$ , the real frequency and fuel flux are always negative, even when  $\omega_{*I}$  is equal or slightly bigger than 0, as we have seen in simulations.

To demonstrate the fuel pinch analytically, we reduce Eq. (2) to the first order of  $\omega_{D\sigma} / \omega$ , and get a form of  $R_\sigma$  similar to Ref. 5

$$R_\sigma = f_\sigma \tau_\sigma \left[ \frac{\omega_{*\sigma} - \omega_{D\sigma}}{\omega} - \frac{\omega_{D\sigma} \omega_{*p\sigma}}{\omega^2} \right]. \quad (4)$$

The following conditions are generally satisfied in a tokamak plasma, and are assumed in this study: (i)  $\omega_{*Ti} > 0$ , meaning the main ion temperature is higher toward the core. (ii) The theory allows that for some tokamak profiles, such as in ITER,<sup>19</sup>  $\omega_{*i}$  could be negative. However, we require  $\omega_{*pi} = \omega_{*i} + \omega_{*Ti} > 0$ , so the temperature gradient should dominate. (iii) The fuel ion density should concentrate at the edge, therefore  $\omega_{*I} < 0$ . (iv) Since the fuel ions are cold, its temperature gradient should be small compared to its density gradient, hence we require that  $\omega_{*pI} = \omega_{*I} + \omega_{*TI} < 0$  and  $T_I < T_i$ . Conditions (iii) and (iv) are essential for fueling. We can simply assume  $\omega_{*TI} = 0$ , as in simulations the fuel ion temperature is set to be the main ion temperature at the outer boundary point.

Define  $\omega_d \equiv \tau_\sigma \omega_{D\sigma}$ ,  $\bar{\omega}_{*\sigma} \equiv \tau_\sigma \omega_{*\sigma}$ ,  $\bar{\omega}_N \equiv (1 - \epsilon) \bar{\omega}_{*i} + \epsilon \bar{\omega}_{*I}$ ,  $\omega_p \equiv (1 - \epsilon) \omega_{*pi} + \epsilon \omega_{*pI}$ , then from Eq. (1) we have  $\text{Im}(1/\omega^2) = [(\bar{\omega}_N - \omega_d) \text{Im}(1/\omega) + \delta] / (\omega_d \omega_p)$ . With  $\text{Im}(1/\omega) = -\gamma / |\omega|^2$  where  $\gamma$  is the linear growth rate, the  $-\text{Im}(R_I)$  term in Eq. (3) becomes

$$-\text{Im}(R_I) = \frac{\epsilon \omega_{*pI}}{\omega_p} \delta - \omega_d \left( 1 - \frac{\omega_{*pI}}{\omega_p} \right) \epsilon \frac{\gamma}{|\omega|^2} + \left( \bar{\omega}_{*I} - \frac{\omega_{*pI}}{\omega_p} \bar{\omega}_N \right) \epsilon \frac{\gamma}{|\omega|^2}. \quad (5)$$

Since  $\epsilon$  is small, it is reasonable that  $\omega_p > 0$ . With the negative  $\omega_{*pI}$ , the first term on the right-hand side of Eq. (5), which is proportional to  $\delta$ , is therefore negative, meaning a nonadiabatic electron out flux should enhance fueling. The second term is proportional to  $\omega_d$  which comes from the toroidal geometry of the device, and this term is also negative, so toroidal effects are also favorable for natural fueling. In the limit of  $\omega_{*TI} = 0$ , the third term becomes  $\epsilon(1 - \epsilon) \omega_{*I} \tau_i (\omega_{*pi} \tau_I / \tau_i - \omega_{*i}) \gamma / (|\omega|^2 \omega_p)$ , which is also negative since  $\omega_{*I} < 0$  and  $\tau_I / \tau_i > 1$  for cold fuel. We conclude that the fuel flux must be naturally negative and going inward toward the core. In this case with only two ion species, the fuel ion goes in at the expense of hot ions going out due to quasineutrality. However, when helium ash is present, the main DT pinch as the helium ash goes outward. Fueling is then useful to maintain the main DT profile, and balance the outgoing helium ash flux.

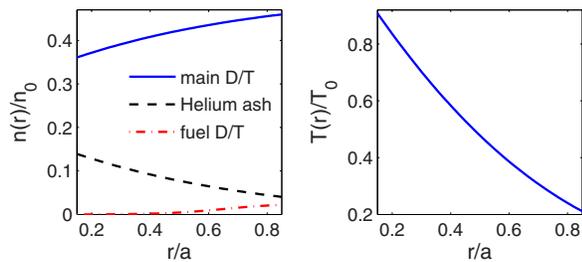
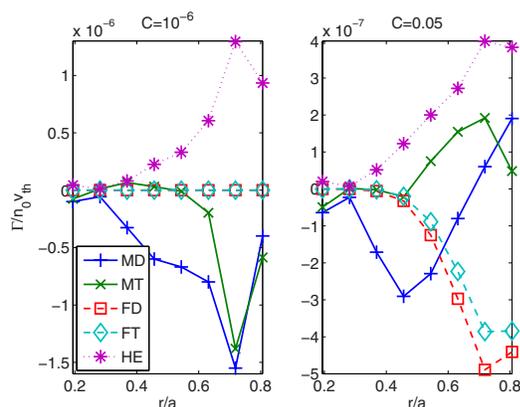


FIG. 5. (Color online) Profiles of the ITER-like case.

The global version of the Cyclone base case presented here have monotonically decreasing density profiles that peak at the magnetic axis. So far there is no general consensus on the ITER density profile. One rather conservative study<sup>19</sup> shows ITER *H*-mode profiles with a completely flat density. The inward peaking helium ash then causes the main DT density to be hollow, or peak near the edge. We now examine the natural fueling mechanism in this situation. The next set of simulations have the same physical parameters as in the Cyclone base case, but use the ITER-like hollow DT density and temperature profiles, as shown in Fig. 5.

The simulation results for this flat density ITER-like case are shown in Fig. 6. This completely flat density profile is overall much less unstable and as a result the cold fuel flux is also lower. This reduces the travel length of the fuel to about 0.8 m before they are heated by ion-ion thermal equilibration. The outward going helium ash flow still causes the main DT to go inward without fueling, but there is a different phenomenon. The main deuterium flux now separates from the main tritium flux. Similar to the previous case, adding cold DT fuel reduces the inward particle pinch of the main DT. However, due to the separation of the main D and T, it is very easy to overshoot with too large a  $C$ , thereby, causing the main tritium to go outward. This overshoot is shown in the right panel of Fig. 3. Given this effect of DT separation in the ITER-like case, one may consider using different density profiles for the deuterium fuel and tritium, possibly less tritium and more deuterium, but it is beyond the scope of this letter and will require future investigations.

FIG. 6. (Color online) The ITER-like case: the particle fluxes for  $C=10^{-6}$  and 0.05.

Another significant new result of the ITER-like case is that the linear growth rate of the ITG-induced instability is reduced by natural fueling.  $\gamma/\Omega_i=3.3, 3.1, 2.5,$  and  $1.9 \times 10^{-4}$  for  $C=10^{-6}, 0.01, 0.03,$  and  $0.05$ . This bonus is due to the hollow density profile of the main DT. In general, the linear growth rate  $\gamma$  from Eq. (1) is roughly  $\sqrt{\omega_d \omega_{*pi}}$  with fueling and  $\sqrt{\omega_d \omega_{*pi}}$  for with main ions only. Since  $0 < \omega_p < \omega_{*pi}$ , fueling decreases  $\gamma$  and hence is a stabilizing effect. This effect is insignificant for the Cyclone base case: From  $C=10^{-6}$  to  $C=0.1$ ,  $\gamma$  is only reduced by 2%, at  $6.8 \times 10^{-4} \Omega_i$ . However, in the ITER-like case, since  $\omega_{*ni}$  is negative, for comparable temperature profiles,  $1/\omega_{*pi}$  is bigger than the Cyclone base case and hence  $\omega_p/\omega_{*pi}=1-\epsilon + \epsilon \omega_{*pi}/\omega_{*pi}$  is smaller. Therefore  $\gamma$  is reduced more by fueling. As a result, the heat fluxes of both main DT and helium ash are also reduced. For  $C=0.05$  in the ITER-like case, the heat fluxes are reduced by nearly 50% from  $C=10^{-6}$ . Natural fueling requires a two-component plasma and relies on weak plasma collisionality. Ion-ion collisions equilibrate the cold fuel and hot core ions. In addition, charge-exchange collisions limit the temperature difference between the cold plasma source and bulk plasma for pellet injection as well as for other fueling scenarios.

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