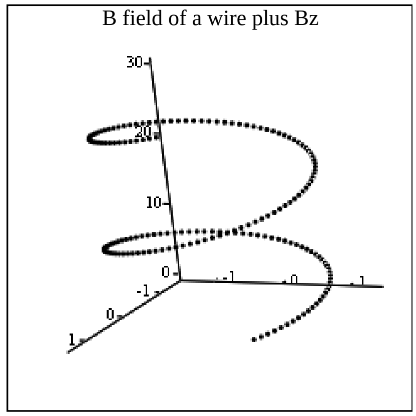
Plotting magnetic fieldlines

What is the trajectory of a magnetic field line? A ball follows a trajectory x(t), y(t), z(t). A field line has no time dependence, it is "just there." For a plot, we need to replace the variable t with the distance along the line s. Then the line can be expressed as x(s), y(s), and z(s). As we vary s, these functions tell us where we will find the point on the field line corresponding to that value of s.



$\left(M1^{\langle 1\rangle}, M1^{\langle 2\rangle}, M1^{\langle 3\rangle}\right)$

At each point in space the B field points in a direction with components B_x, B_y and B_z. So if we move along the B field line a small distance ds, then the end of the B field line moves from (x, y, z) to (x + dx, y + dy, z + dz). The fraction of the distance ds that is in the x direction is B_x / |B|. So if we move along the line a distance ds, then x progresses by (B_x/|B|) ds. So now we see that the distance of x(s) from the starting point is the integral of (B_x/|B|) ds. The expression for x(s) is

$$x(s) = x_0 + \int_0^s \frac{B_x ds'}{|B|}$$ where ds' is a dummy variable.

The B field of a wire

If a straight wire is in the z direction, then the B field is in the θ direction. The x component of a unit vector in the θ direction is -y/r and the y component is x/r. So a unit vector in the magnetic field direction is U = [-(B_θ/|B|)(y/r), (B_θ/|B|)(x/r), 0]. Now we just integrate these with ds. Keep in mind that B_θ is a function of r: $B_\theta(r) = \mu I / 2\pi r$. Let's pretend that μ_0 is one so that we don't need scientific notation for our answers..

$I := 5$ is the current. We will let Bz be zero for now: $Bz := 0$ $\mu_0 := 1$

The vector X will contain the starting point for the line:

Our starting point will be $X := \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ X_0 is x, X_1 is y, X_2 is z.

In terms of the components of X, the radius is $r(X) := \sqrt{(X_0)^2 + (X_1)^2}$

The θ field is $B\theta(X) := \dfrac{\mu_0 \cdot I}{2 \cdot \pi\, r(X)}$

The absolute value of B is $B(X) := \sqrt{B\theta(X)^2 + Bz^2}$

The components of the unit vector along B

$$Ux(X) := \frac{(-X)_1 \cdot B\theta(X)}{r(X) \cdot B(X)} \qquad Uy(X) := \frac{X_0 \cdot B\theta(X)}{r(X) \cdot B(X)} \qquad Uz(X) := \frac{Bz}{B(X)}$$

Below put the three functions to be integrated into the 3-vector Derivs.

$$Derivs(s, X) := \begin{pmatrix} Ux(X) \\ Uy(X) \\ Uz(X) \end{pmatrix}$$

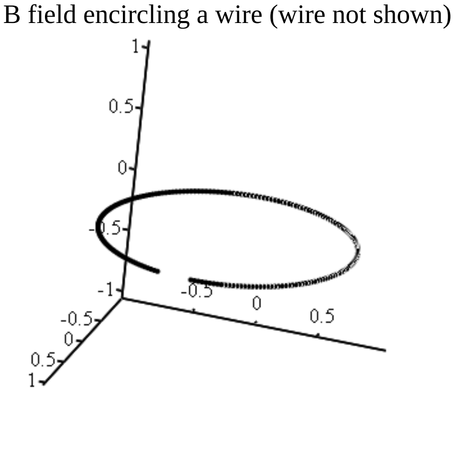
We will put lots of points in our lines. $npoints := 200$

The program "rkfixed" below uses the Runge-Kutta method to integrate the derivatives Derivs a distance of 6 units and divides this into npoints intervals. The B field line is a circle, of course, and it does not quite close because 6 is not quite 2π.

$M0 := rkfixed(X, 0, 6, npoints, Derivs)$ The matrix M will contain s, x(s), y(s) and z(s)

We only need to plot columns 1, 2, and 3. Column zero is not needed.

B field encircling a wire (wire not shown)



$\left(M0^{\langle 1 \rangle}, M0^{\langle 2 \rangle}, M0^{\langle 3 \rangle}\right)$

M0 =

	s	x(s)	y(s)	z(s)
	0	1	2	3
0	0	1	0	0
1	0.03	1	0.03	0
2	0.06	0.998	0.06	0
3	0.09	0.996	0.09	0
4	0.12	0.993	0.12	0
5	0.15	0.989	0.149	0
6	0.18	0.984	0.179	0
7	0.21	0.978	0.208	0
8	0.24	0.971	0.238	0
9	0.27	0.964	0.267	0
10	0.3	0.955	0.296	0
11	0.33	0.946	0.324	0
12	0.36	0.936	0.352	0
13	0.39	0.925	0.38	0
14	0.42	0.913	0.408	0
15	0.45	0.9	0.435	...

You can grab the axes and tilt this 3-d plot.

Try it: Does the gap in the ring close if you make npoints bigger?

The B field of a straight wire PLUS a constant field

Now suppose we do this again and add some B_z. This will cause the B field line to "move" along the z axis as it goes around in the θ direction. The result is a helix.

$Bz := 1$ I have to repeat the formulas above so that they are evaluated with the new B_z.

$B(X) := \sqrt{B\theta(X)^2 + Bz^2}$ $s := 0$ We will start at: $X := \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

The components of the unit vector are:

$Ux(X) := \dfrac{(-X)_1 \cdot B\theta(X)}{r(X) \cdot B(X)}$ $Uz(X) := \dfrac{Bz}{B(X)}$ $Uy(X) := \dfrac{X_0 \cdot B\theta(X)}{r(X) \cdot B(X)}$

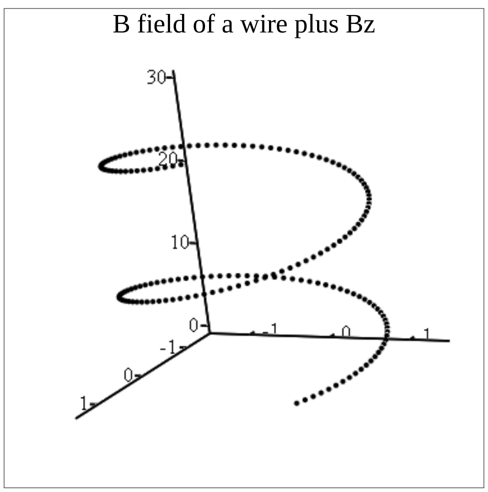
$Derivs1(s, X) := \begin{pmatrix} Ux(X) \\ Uy(X) \\ Uz(X) \end{pmatrix}$

Derivs1 is the 3-vector of derivatives. Integrate them with rkfixed.

M1 is the matrix of answers

$M1 := rkfixed(X, 0, 40, npoints, Derivs1)$

It really is a helix:



$\left(M1^{\langle 1 \rangle}, M1^{\langle 2 \rangle}, M1^{\langle 3 \rangle}\right)$

s x(s) y(s) z(s)

M1 =

	0	1	2	3
0	0	1	1	0
1	0.2	0.928	1.067	0.174
2	0.4	0.852	1.129	0.349
3	0.6	0.772	1.185	0.523
4	0.8	0.688	1.236	0.697
5	1	0.601	1.28	0.872
6	1.2	0.51	1.319	1.046
7	1.4	0.418	1.351	1.22
8	1.6	0.323	1.377	1.394
9	1.8	0.227	1.396	1.569
10	2	0.13	1.408	1.743
11	2.2	0.032	1.414	1.917
12	2.4	-0.066	1.413	2.092
13	2.6	-0.164	1.405	2.266
14	2.8	-0.261	1.39	2.44
15	3	-0.357	1.369	...

You can grab the corners of the 3-d plot and rotate it. Then you can really tell it is a helix.

Try it: What happens if you change Bz to 0.1?

Autoscale for the z axis must remain turned off to see any difference.