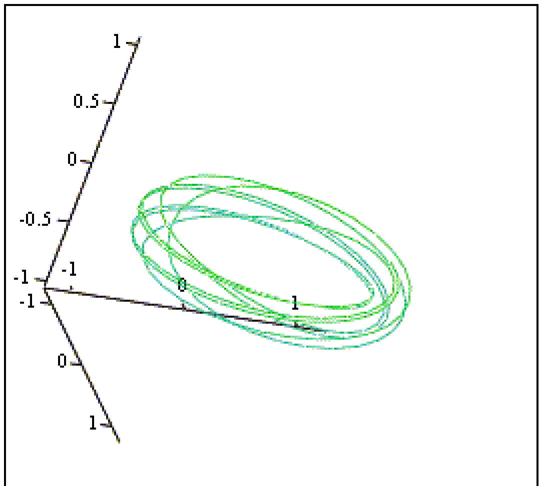
B field of the tokamak



The B lines map a toroidal surface

(x, y, z)

The donut-shaped tokamak is the most successful plasma confinement device. There is a set of toroidal field coils that makes a B_θ. This field is the same one that would be made by a wire on the z axis. A second field is made by the current in the plasma itself. This field is similar to the field of a current loop. The difference is that the current loop has the current concentrated on the minor axis (the center of the donut-shaped tube) and the plasma has the current distributed throughout. We will use the current loop to mimic the plasma current and the field of a straight wire along the z axis to mimic the field of the toroidal coils. The field of a loop is presented in the previous B field of a loop exercise. We will follow a field line and see where it goes.

For the field of the loop, we will need some special functions:

Complete elliptic integral of first kind — of the second kind

$$K(k) := \int_0^1 \frac{1}{\sqrt{(1-t^2)\cdot(1-k^2\cdot t^2)}}\,dt \qquad E(k) := \int_0^1 \sqrt{\frac{1-k^2\cdot t^2}{1-t^2}}\,dt$$

where, for the loop of wire, k is defined in terms of the loop radius a and the cylindrical coordinates r,z at our location X. The vector X will contain the three components (r, θ, z).

The radius of the loop is : $a := 1$

The argument of the elliptic integrals is k which is a function of r, z and a.

$$k(X,a) := \sqrt{\frac{4\cdot X_0\cdot a}{(X_0+a)^2+(X_2)^2}}$$

Let's avoid answers in scientific notation by replacing $\mu/2\pi$ by simply I.

Also set the toridal current to a small value. $I := .1$

In Mathcad notation, the fields from the toroidal current are

$$Br(X) := \frac{I \cdot X_2}{X_0 \cdot \sqrt{\left(X_0 + a\right)^2 + \left(X_2\right)^2}} \cdot \left[-K(k(X,a)) + E(k(X,a)) \cdot \frac{\left[a^2 + \left(X_0\right)^2 + \left(X_2\right)^2\right]}{\left(a - X_0\right)^2 + \left(X_2\right)^2}\right]$$

$$Bz(X) := \frac{I}{\sqrt{\left(X_0 + a\right)^2 + \left(X_2\right)^2}} \cdot \left[K(k(X,a)) + E(k(X,a)) \cdot \frac{\left[a^2 - \left(X_0\right)^2 - \left(X_2\right)^2\right]}{\left(a - X_0\right)^2 + \left(X_2\right)^2}\right]$$

The field above is in the r,z plane and is called the poloidal field. To this we need to add the toroial field made by a wire along z.

We must make the current for the toroidal field very much larger than the current in the plasma. [The toroidal coils have many turns which increases their effective current.]

$$Itor := 5$$

Again we ignore $\mu/2\pi$. Bθ is defined as:

$$B\theta(X) := \frac{Itor}{X_0}$$

We will need the absolute value of the field

$$B(X) := \sqrt{Br(X)^2 + Bz(X)^2 + B\theta(X)^2}$$

DY3 will be the three vector components of the unit vector along B. Note that the length in the θ direction is r dθ. So to find dθ, I must divide B_θ / |B| by r and r is the zeroth component of X.

$$DY3(s,X) := \begin{pmatrix} \dfrac{Br(X)}{B(X)} \\ \dfrac{B\theta(X)}{B(X) \cdot X_0} \\ \dfrac{Bz(X)}{B(X)} \end{pmatrix}$$

Components
r
θ
z

Our starting point for the field line integration will be 0.15 units above the current ring, at a radius of 1. The starting value of θ does not matter.

$$X := \begin{pmatrix} 1 \\ 0 \\ 0.15 \end{pmatrix}$$

The circumference of the torus is 6.3 units so we will go a distance of 50 units (that is about 8 times around the torus the long way). We will divide this into 200 increments.

$npoints := 200$ $\qquad$ $M4 := rkfixed(X, 0, 50, npoints, DY3)$

We integrate using a "dummy variable" ds which is the distance along the field line.

s r θ z

rix of answers.

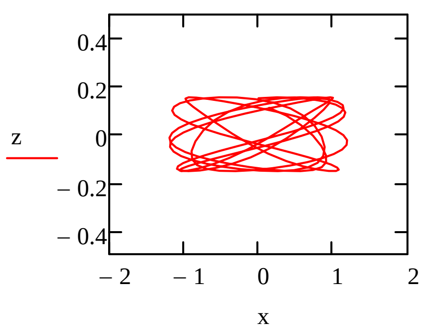
M4 =

	0	1	2	3
0	0	1	0	0.15
1	0.25	1.031	0.244	0.154
2	0.5	1.061	0.481	0.152
3	0.75	1.088	0.713	0.146
4	1	1.111	0.939	0.136
5	1.25	1.132	1.161	0.123
6	1.5	1.15	1.379	...

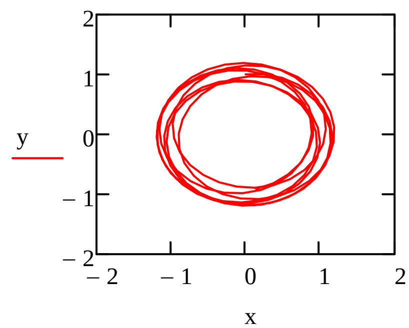
We will convert r,θ,z to x,y,z for plotting $i := 0 .. npoints$

$x_i := M4_{i,1} \cdot \sin(M4_{i,2})$ $y_i := M4_{i,1} \cdot \cos(M4_{i,2})$ $z_i := M4_{i,3}$

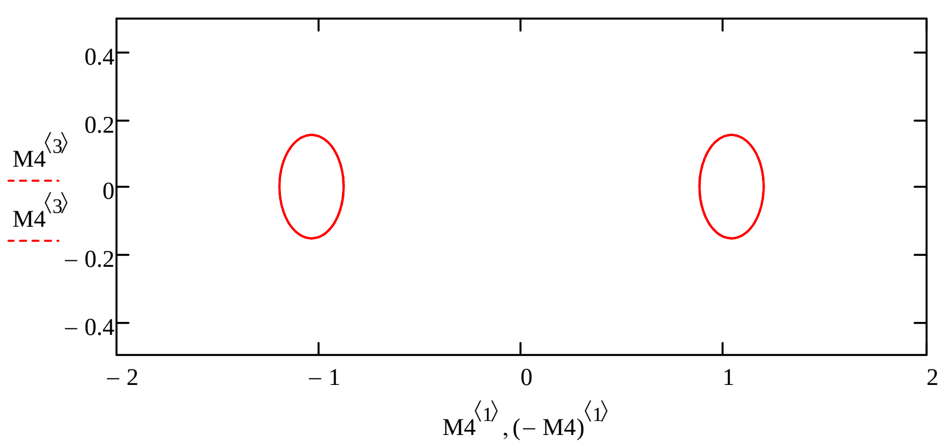
Below is a view from the side of the spiralling field line.



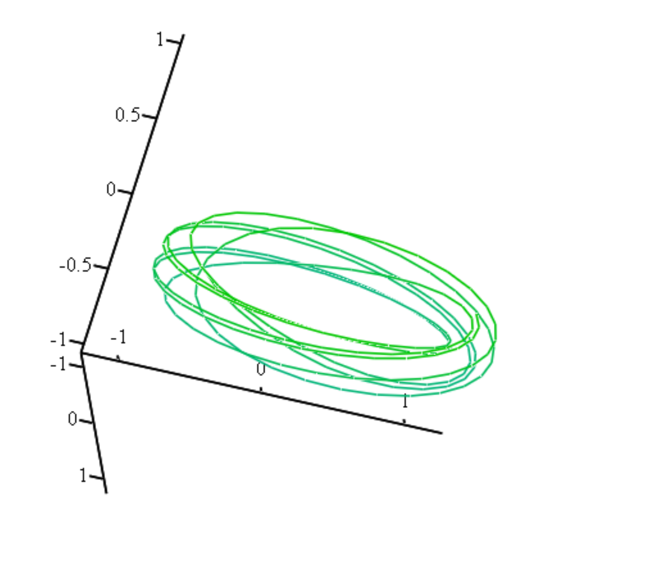
Below is a view from the top looking down. This plot shows the donut hole more clearly.



Below is the path followed by the field line projected onto the r,z plane.



Below is a 3-d plot of the path of a field line.You can click on a corner and rotate it.



The B lines map a toroidal surface

(x, y, z)

In the r,z plane, the field line makes a circle around the minor axis.
If the field line is followed far enough, it maps out donut-like surface. A characteristic of the tokamak device is that it has **nested flux surfaces** mapped by the field lines. A gyrating particle never moves far from the flux surface it begins upon.

The **q value** of a flux surface is defined as the number of times the field line goes around the long way when going around once the small way.

Try it: If the loop current is increased, does the **q value** get bigger or smaller?

Now let's pick a field line a little further from the minor axis (the location of the current loop). This makes our donut look fatter. Tokamaks with smaller holes in the middle are currently a topic of research. The field lines on the inside of the donut (smaller r) spiral more loosely than those on the outside (bigger r).

The new starting point:

$$X := \begin{pmatrix} 1 \\ 0 \\ 0.4 \end{pmatrix}$$

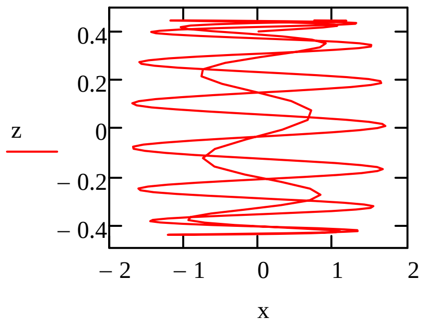
The new plot:

$$M4 := \mathrm{rkfixed}(X, 0, 100, npoints, DY3)$$

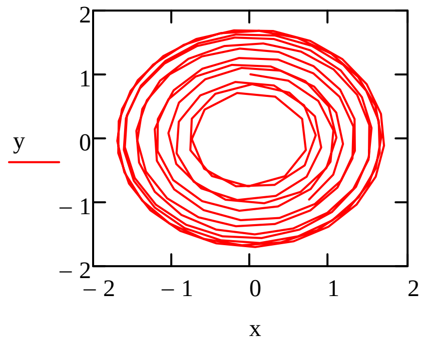
We will convert r,θ,z to x,y,z for plotting

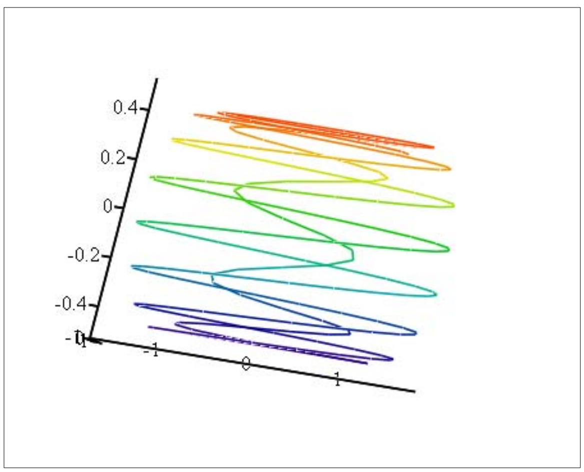
$$x_i := M4_{i,1} \cdot \sin(M4_{i,2}) \qquad y_i := M4_{i,1} \cdot \cos(M4_{i,2}) \qquad z_i := M4_{i,3}$$

Below is a view from the side of a spiralling field line.



Below is a view from the top looking down.

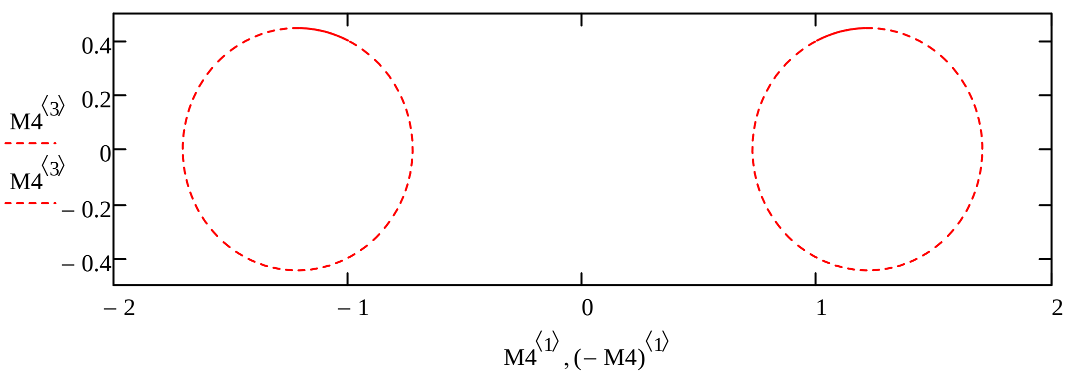




(x, y, z)

On the left is a 3-d plot of the path of a field line. You can click on a corner and tilt the plot.

Below is the path followed by the field line in the r,z plane.



A tokamak with a small donut hole is called a **small aspect ratio tokamak.** The aspect ratio is the distance from the major axis divided by the distance from the minor axis and is usually evaluated at the plasma boundary.